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A MODEL OF THE QUIET IONOSPHERE

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SUMMARY

Analysis of high-altitude-rocket electron and ion density measurements suggests a simple model of the quiet ionosphere. Near $h_{\max} F_2$ the profile is given by the " α -Chapman" function, that is, the Chapman electron density equation with $\sec \chi = 1$ and the scale height H a constant. A method is given for determining $h_{\max} F_2$, $N_{\max} F_2$, and H from $N(h)$ data obtained from ionograms. Well above the peak, the profile is taken to have a constant exponential slope of 200 km during the day and 150 km at night. If simultaneous nearby measurements of the total electron content are available, a more accurate slope may be computed to provide the profile up to about 1000 km altitude.

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INTRODUCTION

Electron density profiles above the F_2 maximum obtained with rockets and satellites are summarized by Wright in Reference 1. He found that he could obtain an approximate agreement with these results if he used the Chapman electron density equation with scale height H considered to be constant at 100 km and with $\sec \chi = 1$. He computed the ratio of electron density above the maximum to that below; but these values tended to be a little higher than the measured values, which were obtained mainly from orbiting satellites and military rockets where accuracy was "not of the best." Subsequently Berning (Reference 2) obtained a sunrise profile to 1500 kilometers which, when normalized, did not agree well with previous profiles. His results indicated the existence of a considerable gradient in the electron scale height; however, this profile was—due to unforeseen circumstances—obtained at ground sunrise when conditions could be changing at a rapid rate. A little later, Bowles (Reference 3) published a daytime profile obtained by the incoherent backscatter technique, which indicated an approximate Chapman distribution with $H \approx 90$ km. His night profile indicates $H \approx 75$ km, although the densities close to the maximum are somewhat larger than a Chapman distribution with this scale height.

Nisbet and Bowhill (Reference 4) published a series of profiles obtained with military rockets under difficult scientific conditions. They attempted to compare their normalized results with a Chapman distribution by utilization of a variable neutral scale height given by Kallman in Reference 5; the agreement was not particularly good. Berning[†] revealed profiles showing a constant scale height H well above $h_{\max} F_2$ of about 100 km in the daytime and 72 km during the evening.

Hanson and McKibbin (Reference 6) reported ion density measurements which gave an H value of 75 km in the evening for altitudes well above the maximum density. Pineo et al.[‡] reported on daytime results obtained by incoherent backscatter techniques, which showed a scale height gradient similar to the sunrise results of Berning. More recently, Jackson and Bauer (Reference 7) obtained a daytime profile with H also given as 100 km.

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†Berning, W. W., "Experimental Measurement of Electron Temperature and Electron and Ion Density by a Direct Measurement and a Propagation Technique." Paper presented at Union Radio Scientifique Internationale Meeting, Washington, May 1961.

‡Pineo, V. C., et al., "Expected Studies of F-Region Ending in Coherent Back Scattering Technique at Frequencies Around 400 Mc." Paper presented at Union Radio Scientifique Internationale Meeting, Washington, May 1961.

This paper discusses the electron and ion density results obtained with rockets under quiet ionospheric conditions and reasonably favorable scientific conditions. A simple model of the quiet ionosphere, which can be expressed approximately in analytical form, is obtained. This model is used to develop a means of obtaining from ionograms an approximate electron density profile and total electron content. It is also shown how such data used in conjunction with total electron content measurements make possible the determination of the electron density profile above $h_{max} F_2$.

DISCUSSION

If the results of Berning*, Hanson and McKibbin (Reference 6), and Jackson and Bauer (Reference 7) are examined near the peak of the F_2 region, it is found that the measured electron densities do not follow a Chapman function with a scale height as high as 100 km. However, these results are all in agreement that, well above the peak altitude, a constant exponential slope exists which—on the assumption that the ion and electron temperatures are equal—corresponds to an H value of about 100 km in the daytime and 75 km in the evening.

Yonezawa (References 8 and 9) showed that under the influence of vertical diffusion and nighttime attachment any $N(h)$ curve will at night tend to take the form of a Chapman layer of constant scale height H and $\sec \chi = 1$, referred to as an " α -Chapman" layer. Long (Reference 10) demonstrated that, if the $N(h)$ analysis of the ionogram includes an allowance for the underlying ionization that is usually neglected, the profiles for all latitudes follow an α -Chapman variation to first order. The present paper shows that, for the limited daytime rocket data available, this also holds for quiet daytime conditions.

Figure 1 gives the Jackson-Bauer results plotted near the F_2 peak. The circled points represent the α -Chapman function with a scale height of 57 km, and the numbers show the number of scale heights above or below the maximum. There is indeed a very close agreement with this Chapman function over the range from -1 scale height up to about $+1.6$ scale heights with $h_{max} = 287$ km. At 380 km a break occurs in the profile slope until, at an altitude of about 400 km, a constant asymptotic slope is reached. This disagrees with the result given by Jackson and Bauer that hydrostatic equilibrium begins at 350 km. Figure 2 shows a portion of Berning's unpublished quiet day results, with equilibrium also beginning at 400 km. His results follow a Chapman distribution very accurately from about -1 to $+1.5$ scale heights with $H = 60$ km and $h_{max} = 280$ km. Figure 3 gives the nighttime ion density results of Hanson and McKibbin, which also show equilibrium beginning at 400 km. Their results follow a Chapman distribution from below -1.2 to $+1.3$ scale heights with $H = 43$ km and $h_{max} = 309.5$ km. Berning's unpublished night flight at Wallops Island, Va., at 2143 EST July 13, 1960, also follows the Chapman function very accurately from -2.0 to $+1.8$ scale heights with $H = 63$ km and $h_{max} = 357$ km.

It is difficult to state precisely at what altitude the neutral particle scale height has a value equal to H , but it would seem reasonable to suppose that it is near $h_{max} F_2$. Kallmann's (Reference 11)

*See footnote, page 1.

Figure 1—Chapman distribution fitted to Jackson and Bauer electron density profile. (NASA 8.10; April 27, 1960, 1502 EST Wallops Island, Va.)

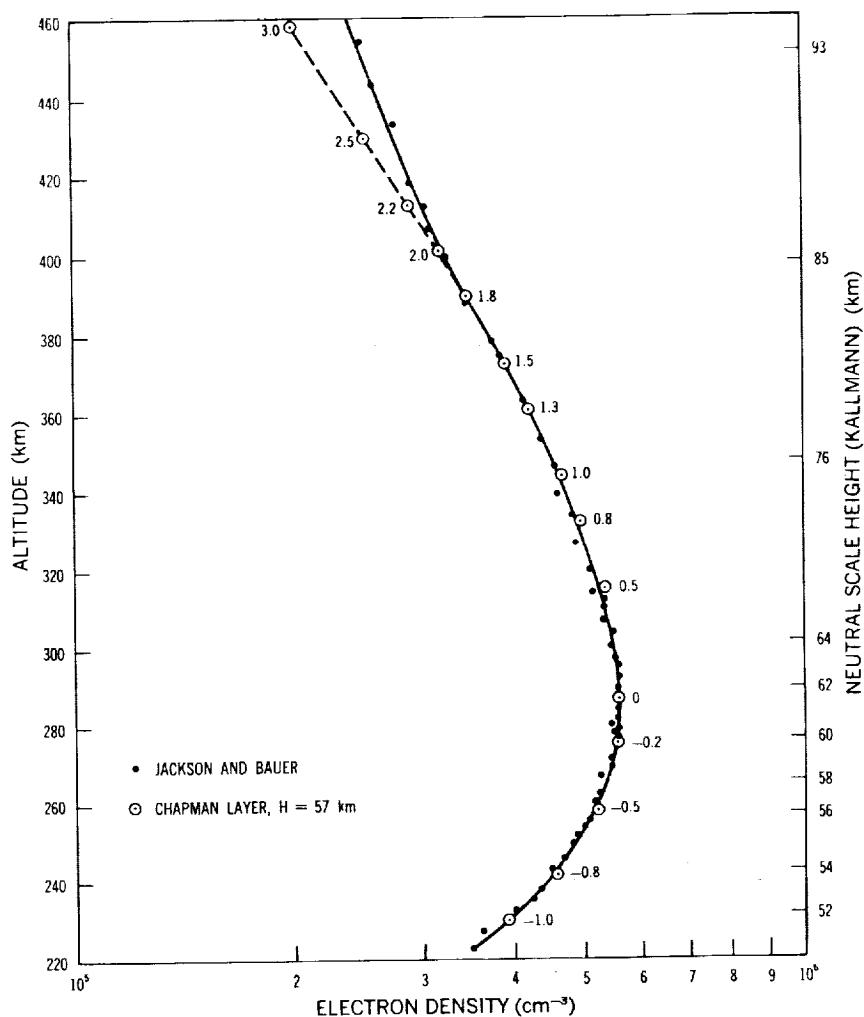
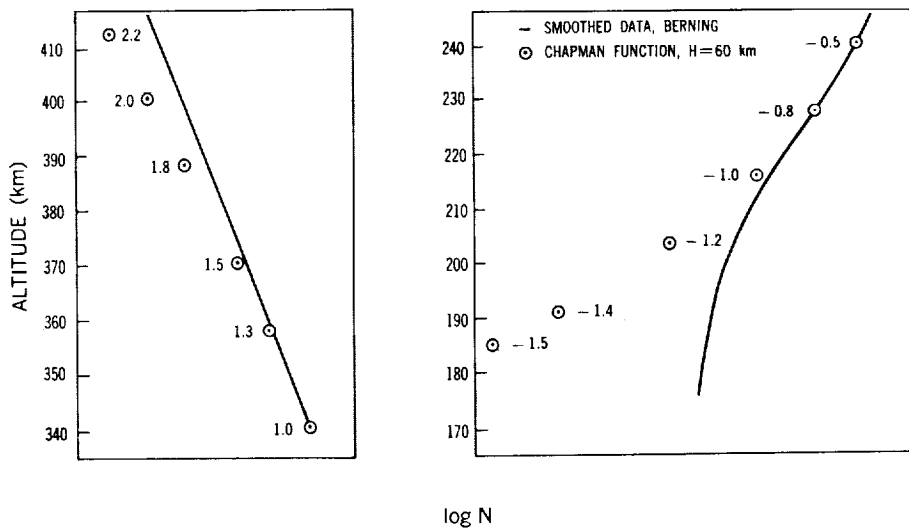


Figure 2—Chapman distribution fitted to Berning electron density profile. (OB 11.03 Strong Arm III; July 13, 1960, 0947 EST Wallops Island, Va.)



average daytime scale heights are shown on the right-hand side of Figure 1; the agreement is even better when compared with Berning's daytime flight. Kallmann's nighttime average value agrees reasonably well with Berning's summer night firing, but rather poorly with Hanson and McKibbin's November night firing.

These data thus indicate that under quiet conditions the F_2 is α -Chapman from -2.0 to $+1.5$ scale heights at night and from -1.0 to $+1.5$ in the daytime. Above 1.5 scale heights there is a transition region of about 25 kilometers after which diffusive equilibrium exists, with a constant exponential slope of about 150 km in the evening and 200 km in the daytime. Data late at night are not available, and it is likely that before sunrise the slope may decrease to lower values; a method will be suggested to check on this.

MODEL FOR THE QUIET MIDDLE-LATITUDE IONOSPHERE

Since the ionosphere decreases the tracking accuracy of systems using radio waves, it would be desirable to express the density profile in terms of approximate analytical functions to simplify calculations. Therefore the ionization in the E-region and above was reviewed, and a reasonable approximation in the daytime below -1.0 scale height may be had by drawing a straight line on semilog paper to $N_{max}(E)$ at an altitude of 100 km. While the actual profiles indicate variations around this assumed profile, the electron contents are about equal. Figure 4 shows the approximate model for the quiet middle-latitude ionosphere.

Table 1 presents an approximate representation of the profiles in analytical form, which may be useful in refraction calculations; S is obtained from a knowledge of the initial and final values. Also given are formulas for the electron content. It is assumed here that the nighttime region is entirely Chapman below h_{max} . If appreciable sporadic-E exists, it may be assumed to have a plasma frequency equal to the lowest F-reflection frequency plus one-half the gyrofrequency and an average thickness of 1 km (Seddon, Reference 12).

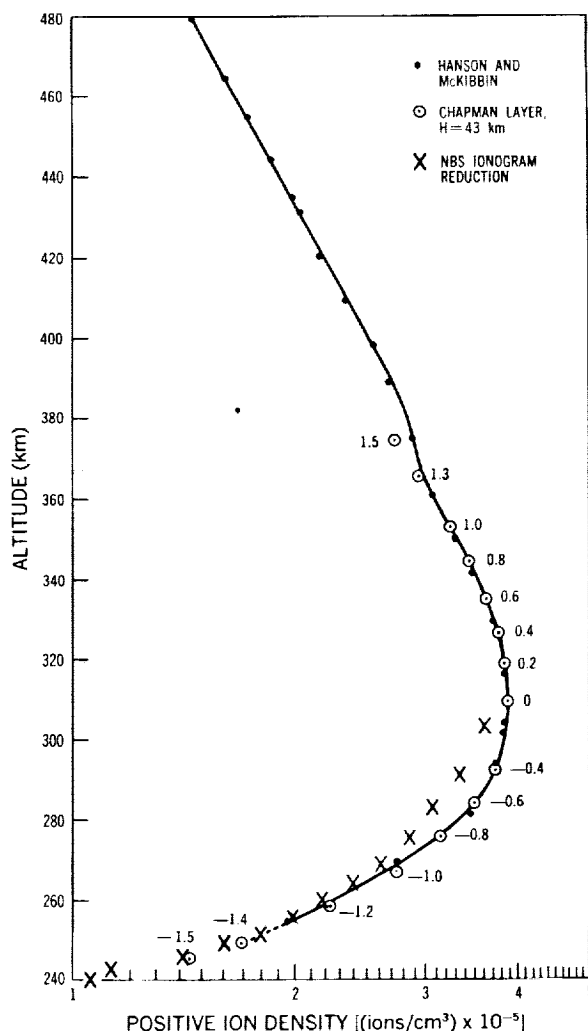


Figure 3—Chapman distribution fitted to Hanson and McKibbin ion density profile. (Nov. 9, 1960, 2044 EST Wallops Island, Va.)

Table 2 gives the electron content calculated from the formulas of Table 1 as compared with the values obtained from numerical integration of rocket data, with the assistance in some cases of E-region data obtained from the P'-f analysis performed by the National Bureau of Standards.

Comparison of the rocket results and the results obtained by the National Bureau of Standards using ionograms from a nearby ionosonde showed that the agreement was generally good but that considerable disagreement existed at times for H and/or h_{max} . The National Bureau of Standards technique involves a parabolic assumption near h_{max} that should give fairly good results if the ionogram can be read to frequencies very close to f_oF_2 . Below about $-1/2$ scale height, however, the parabolic

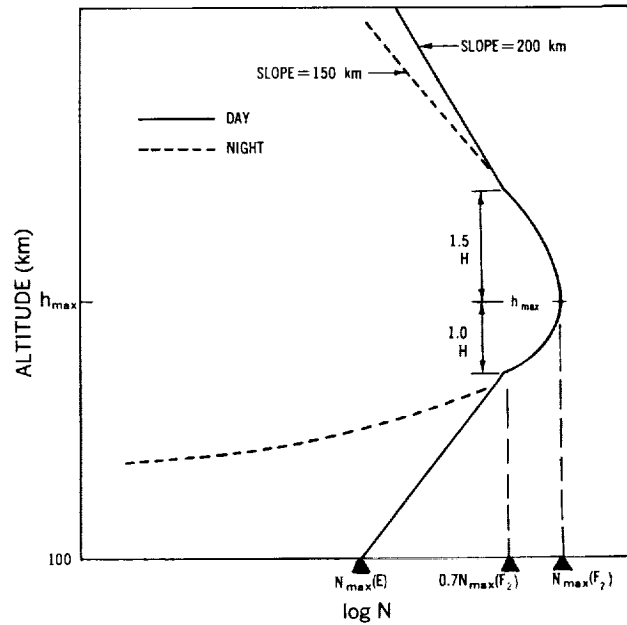


Figure 4—Simple model of the quiet middle-latitude ionosphere

Table 1

Analytical Representation of the Model

Night N		$N_{max}(F_2) \text{ Ch}(z)$	$z \leq 1.5, z = \frac{h - h_{max}}{H}$
		$0.7N_{max}(F_2) e^{-\frac{h - h_{max} - 1.5H}{150}}$	$h \geq h_{max} + 1.5H$
Day N		$N_{max}(E) e^{\frac{h - 100}{S}}$	$N_{max}(E) \leq N \leq 0.7N_{max}(F_2),$ $100 \leq h \leq h_{max} - H$
		$N_{max}(F_2) \text{ Ch}(z)$	$-1 \leq z \leq 1.5$
		$0.7N_{max}(F_2) e^{-\frac{h - h_{max} - 1.5H}{200}}$	$h \geq h_{max} + 1.5H$
Night	N_b	$1.312 HN_{max}(F_2)$	
	N_a	$1.30 HN_{max}(F_2) + 150 \times 0.7N_{max}(F_2)$	
	N_t	$2.61 HN_{max}(F_2) + 150 \times 0.7N_{max}(F_2)$	
Day	N_b	$S[0.7N_{max}(F_2) - N_{max}(E)] + 0.88 HN_{max}(F_2)$	
	N_a	$1.30 HN_{max}(F_2) + 200 \times 0.7N_{max}(F_2)$	
	N_t	$S[0.7N_{max}(F_2) - N_{max}(E)] + 2.18 HN_{max}(F_2) + 200 \times 0.7N_{max}(F_2)$	

Table 2
Summary of Results

Date-Time-Place*	Data Source	H (km)		h _{max} (km)		N _{max} (10 ⁵ cm ⁻³)		N _b (10 ¹² cm ⁻²)	Calculated† N _b	N _a (10 ¹² cm ⁻²)	Calculated‡ N _a	N _a /N _b	Calculated† N _a /N _b
		Parabolic	Chapman	Observed	Chapman	Observed	Chapman						
1	Rocket	—	57.0	287.0	287.0	5.60	5.60	5.87	6.00	11.85	12.00	2.02	2.0
2	P ¹ -f	59.3	60.3	299.0	294.0	5.64	5.57	6.51, 6.24†	6.15	—	12.16	—	2.0
3	P ¹ -f	49.7	59.0	281.0	284.3	5.10	5.13	5.37, 5.55†	5.60	—	11.11	—	2.0
4	Rocket	—	63.0	357.0	357.0	4.60	4.60	3.91	3.80	8.60	8.59	2.20	2.26
5	P ¹ -f	49.6	63.0	363.0	370.7	4.54	4.605	3.27, 3.60†	3.81	—	8.60	—	2.26
6	Rocket	—	43.0	310.0	309.5	3.90	3.90	2.14	2.20	6.89	6.27	3.22	2.9
7	P ¹ -f	43.9	50.5	318.0	321.7	3.72	3.76	2.34, 2.48†	2.49	—	6.41	—	2.6
8	Rocket	—	60.0	280.0	280.0	5.03	5.03	5.77	5.50	10.96	10.96	1.90	2.0

*April 27, 1961: 1 - 1502 EST, Wallops Island
 2 - 1500 EST, Wallops Island
 3 - 1500 EST, Ft. Belvoir
 July 13, 1960: 4 - 2143 EST, Wallops Island
 5 - 2145 EST, Ft. Belvoir
 Nov. 9, 1960: 6 - 2044 EST, Wallops Island
 7 - 2044 EST, Wallops Island
 July 13, 1960: 8 - 0945 EST, Wallops Island

†Calculated from Table 1.

‡Chapman function above N(h) reduced data.

approximation becomes rapidly poorer; and examination of their data frequently shows few or no points higher than $-1/2$ scale height. In addition, the method depends on an accurate value of $f_0 F_2$. The same $N(h)$ data were used to obtain the best fit possible to a Chapman distribution; and these results generally gave better agreement with the rocket results including $N_{max} F_2$, which is obtained without the use of $f_0 F_2$.

EXPERIMENTAL METHOD

While various methods can be devised to fit the $N(h)$ values obtained, the method used in this paper can be performed on a desk computer or an electronic computer. Both computers actually were used, the electronic computer being the relatively slow LG-30 digital computer.

All the values obtained from the $N(h)$ reduction that were above 1 scale height below the estimated h_{max} in the daytime, and above about 1.5 scale heights below h_{max} at night, were used. The highest altitude was called h_1 , the next h_2 , etc. If the points fall on a Chapman curve, the relation

$$H_1 = \frac{h_1 - h_i}{z_1 - z_i} \quad (1)$$

must be constant for each value of i , where z is the solution of $Ch(z)$ with $\sec \chi = 1$; that is,

$$N = N_{max} \exp \frac{1}{2} (1 - z - e^{-z}) \quad (2)$$

The values of z can be found from the relation

$$Ch(z_i) = \frac{N_i}{N_{max}} \quad (3)$$

The computer is given a value of N_{max} a little less than the value computed from the observed $f_0 F_2$. The h values are obtained from the $N(h)$ reduction, along with the corresponding value of N . The result obtained is a series of values H_{12} , H_{13} , etc. which will increase steadily in value if the N_{max} used is too small and if the $N(h)$ values are, in fact, truly Chapman. Because of the fact that there is some variation from a true Chapman function, the variation will not be entirely smooth. The mean value \bar{H} is computed, and also the average deviation from the mean.

A slightly larger value of N_{max} is then given to the computer, and the same computations are made again. If the $N(h)$ points were exactly on a Chapman curve, a value of N_{max} ultimately would be found where the average deviation would be zero. Because of irregular variations from true Chapman, the average deviation decreases rapidly and nearly linearly to a value dependent on the size of such variations and then increases again, as illustrated by Figure 5. The altitude $h_{max} F_2$ is the average of the values obtained for h_{max_i} for the correct N_{max} , where

$$h_{max_i} = h_i - \bar{H} z_i \quad (4)$$

RESULTS

The values obtained for H , h_{max} , and N_{max} are shown in Table 2 in the columns labeled "Chapman," and those obtained by the NBS using the parabolic approximation are shown in the other columns. The table is arranged to facilitate comparison with the rocket results. The Wallops Island ionosonde is only a few miles from the rocket launcher. The Fort Belvoir ionosonde is approximately 160 km to the west, or about 8 minutes earlier in solar time.

The values for both H and N_{max} obtained by the Chapman technique generally agree better with the rocket results than those obtained from the parabolic approximation except for the one case of the winter night firing. It is quite likely that the ion trap measurements are not as accurate as the electron density measurements, since the $N(h)$ reduction disagrees by a variable amount up to nearly 10 percent, which can introduce a considerable change in the value of H .

In all the other cases examined, it was found that the $N(h)$ reductions were in good agreement with the rocket data except that, above 1 scale height below the maximum, the values of N were always slightly lower. However, this slight decrease tends to make both H and $h_{max} F_2$ too large and $N_{max} F_2$ slightly too small. The result causes N_b to be too large in the daytime. A better result is obtained by integrating the $N(h)$ results up to -1 scale height and then using the Chapman profile to peak. At night, the ionization at the lower altitudes not determined by the ionogram reduction technique more than offsets this effect. It is more accurate at night (and easier) to assume that the region is Chapman rather than to integrate the profile obtained by the reduction.

Figure 5 shows some variation in $N_{max} F_2$ obtained by the different measurements. At the time of the rocket firing, the value of $N_{max} F_2$ at Fort Belvoir by the Chapman-fit method was 5.13×10^5 electrons/cm³, whereas the Wallops Island ionosonde gave 5.57×10^5 electrons/cm³. This indicates a horizontal gradient in the electron density, increasing to the east. The rocket firing was approximately east of the ionosonde, and the measured value was 5.60×10^5 electrons/cm³. If the observed $f_o F_2$ at Wallops Island is assumed to be correct, the horizontal gradient would have to be in the opposite direction east of the ionosonde.

Another interesting observation is that the electron density profiles both day and night show a departure from the Chapman profile close to 225 km. Also, if the data from the daytime ionograms are used to calculate $h_{max} F_2 - H$ or, from the nighttime ionograms, $h_{max} F_2 - 2H$, a value around 230 km is usually obtained. It would thus seem that the strong tendency to assume a Chapman distribution exists above about 225 km but not below it.

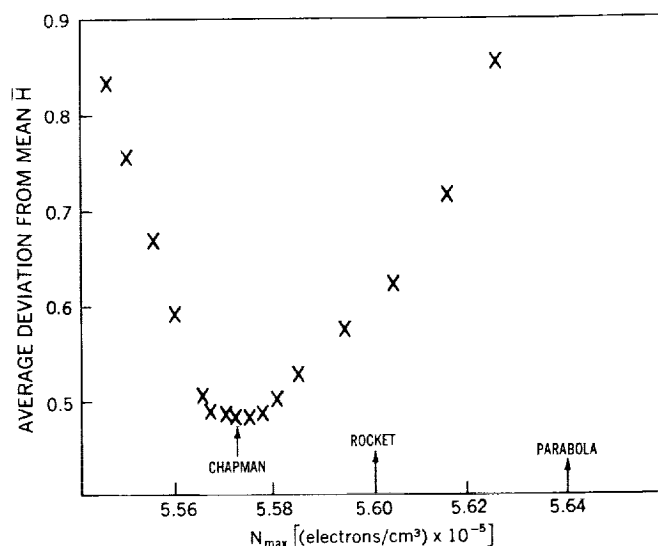


Figure 5— Determination of $N_{max} F_2$ by Chapman-fit to ionogram $N(h)$ reduction. (April 27, 1961, 1500 EST Wallops Island, Va.)

The ionogram data for July 13, 1960, was not included in Table 2. Although the day was not classified as stormy, the F_2 region was undergoing rather rapid changes with time. None of the Fort Belvoir ionograms near the time of the rocket firing agreed with the rocket data. The values obtained for H varied at times at a rate of 1 km per minute; in a few cases only three points were available within 1 scale height of the peak. Under such circumstances one cannot conclude that the profile was truly Chapman. It was noted, however, that the values of f_oF_2 computed on the assumption that it was Chapman gave values that were from 1 to 2 percent larger than the observed f_oF_2 . For these instances the recorded virtual heights were abnormally large, sometimes exceeding 600 km — presumably due to the fact that f_oF_2 was only slightly larger than f_oF_1 . This results in considerable absorption and a rapid rate of change in virtual height with frequency near f_oF_2 , which raises the question of whether it is possible to measure f_oF_2 accurately from such an ionogram. Additional evidence is seen in the fact that during the period 20 minutes before firing time to 80 minutes after — during which ten ionograms were taken — only one ionogram (20 minutes after firing) showed f_oF_2 at Fort Belvoir to be greater than the rocket measured value at Wallops Island, with only two (30 and 40 minutes after firing) being nearly as large.

If the ionogram reduction shows a Chapman distribution, one can obtain a reasonably accurate value for N_b . Then, if measurements are made nearby of the total electron content by means of satellites or two-frequency moon echoes, it is a simple matter to compute the value of the constant exponential slope above $h_{max} F_2$. The rocket values of $N_a + N_b$ were assumed to be known, and the calculation of these slopes agreed quite well in every case. It thus seems feasible to make measurements of the slope by this technique under quiet conditions, provided that it is not done near sunrise or sunset.

The ratios N_a/N_b that were obtained are quite interesting. Spring afternoon, summer morning, and summer evening values are all fairly close to 2.0. Garriott (Reference 13) also finds a similar value for autumn and winter days. On autumn and winter nights, however, he finds the ratio to be as high as 4. His two winter evening measurements made at 2044 local time are in very good agreement with the value of 3.2 reported here for that time on November 9. The model suggested in this paper shows that the ratio will increase if the value of H near h_{max} decreases to lower values with no significant change occurring in the constant logarithmic slope above h_{max} . As the ionosphere closely approximates a Chapman region on the lower side at night, and as the ratio for a true Chapman region is 2.14 (Wright, Reference 1), one would expect that the ratio would be greater than this whenever the value of H is less than one-half the exponential slope, that is, about 75 km in the evening.

Regarding disturbed ionospheric conditions, only a few remarks can be made. Berning* reported on a flight to 400 km at Wallops Island under disturbed conditions. The frequency f_oF_2 was below normal, and $h_{max} F_2$ was higher than normal. The F_2 region was very close to a Chapman distribution from $-1/2$ scale height to the peak of the flight, $+1.3$ scale heights. The value f_oF_1 was nearly equal to f_oF_2 , and under such conditions it is difficult to obtain an ionogram reduction close to the peak. Also, the shape of the profile above $+1.3$ scale heights is not yet known in such cases.

*See footnote, page 1.

CONCLUSIONS

The $N(h)$ reduction of ionograms can be checked easily to determine whether the F_2 region follows an α -Chapman distribution. If it does, the scale height H , the altitude $h_{max} F_2$, and electron content N_o can be determined with more consistent accuracy by a best-fit to a Chapman function than by using the parabolic technique. In addition, it is possible to predict accurately the electron density profile to 1.5 H above $h_{max} F_2$ and to predict approximately the profile above this altitude, and therefore to obtain a fair value for the electron content N_o during the daytime and evening. If the total content is measured by some method, the electron distribution may be predicted to nearly 1000 km by use of ionogram data and the suggested model, with the required logarithmic slope obtained from the total content measurement.

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REFERENCES

1. Wright, J. W., "A Model of the F Region Above $h_{max} F_2$," *J. Geophys. Res.* 65(1):185-191, January 1960
2. Berning, W. W., "A Sounding Rocket Measurement of Electron Densities to 1500 Kilometers," *J. Geophys. Res.* 65(9):2589-2594, Sept. 1960
3. Bowles, K. L., "Incoherent Scattering by Free Electrons as a Technique for Studying the Ionosphere and Exosphere: Some Observations and Theoretical Considerations," *J. Res. Nat. Bur. Standards* 65D(1):1-14, Jan.-Feb. 1961
4. Nisbet, J. S., and Bowhill, S. A., "Electron Densities in the F-Region of the Ionosphere from Rocket Measurements. II. Results of Analysis," *J. Geophys. Res.* 65(11):3609-3614, November 1960
5. Kallmann, H. K., "A Preliminary Model Atmosphere Based on Rocket and Satellite Data," *J. Geophys. Res.* 64(6):615-623, June 1959
6. Hanson, W. B., and McKibbin, D. D., "An Ion-Trap Measurement of the Ion Concentration Profile Above the F_2 Peak," *J. Geophys. Res.* 66(6):1667-1671, June 1961
7. Jackson, J. E., and Bauer, S. J., "Rocket Measurement of a Daytime Electron-Density Profile up to 620 Kilometers," *J. Geophys. Res.* 66(9):3055-3057, Sept. 1961

8. Yonezawa, T., "On the Influence of Electron-Ion Diffusion on the Electron Density and Height of the Nocturnal F2 Layer," *J. Radio Res. Lab.* 2(8):125-136, April 1955
9. Yonezawa, T., "On the Influence of Electron-Ion Diffusion on the Electron Density and Height of the Nocturnal F2 Layer (Supplement)," *J. Radio Res. Lab.* 2(9):281-291, July 1955
10. Long, A. R., "The Distribution of Electrons in the Nighttime Ionosphere," *J. Geophys. Res.* 67(3):989-997, March 1962
11. Kallmann-Bijl, H. K., "Daytime and Nighttime Atmospheric Properties Derived from Rocket and Satellite Observations," *J. Geophys. Res.* 66(3):787-795, March 1961
12. Seddon, J. C., "Sporadic-E As Observed with Rockets," NASA Technical Note D-1043, July 1961
13. Garriott, O. K., "The Determination of Ionosphere Electron Content and Distribution from Satellite Observations; Part 2," *J. Geophys. Res.* 65(4):1151-1157, April 1960

